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Unsteady computations for flow past two oscillating airfoils

V.Ramesh
Computational and Theoretical Fluid Dynamics Division,
National Aerospace Laboratories,
Bangalore, India 560 017, e-mail:vr@ctfd.cmmacs.ernet.in

S.M.Deshpande
Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research,
Bangalore, India 560 065, e-mail:smd@jncasr.ac.in

1 Introduction

In this paper we present the work on the latest developments of a grid free method for computing inviscid unsteady flow past multiple moving bodies. Least Squares Kinetic Upwind Method(LSKUM) [3, 4] is a kinetic theory based grid free scheme for solving the inviscid compressible Euler equations of gas dynamics. LSKUM has been extended to applications with **m**oving **n**odes(LSKUM_MN) [8]. Spatially higher order accuracy is achieved(in LSKUM as well as LSKUM_MN) using the two step defect correction method. In case of LSKUM_MN, it has been shown that defect correction step necceciates the recalculation of moving fluxes [8, 9] at not only all the immediate neighbouring nodes(secondary nodes) but also at the neighbouring points of the secondary nodes. This leads to considerable increase in computational time. In the present work we propose to use the Modified CIR splitting(MCIR) [10, 11] to obtain spatially higher order accuracy in LSKUM_MN. MCIR splitting is a method to achieve spatially higher order accuracy without using the two step defect correction method. Apart form the implementation of MCIR splitting in LSKUM_MN, we have also adopted the weighted least squares approach based on Eigenvector basis [6]. In this approach the least squares approximations for all the derivatives reduce to an equivalent 1-D form. For the unsteady calculations we have used the well known dual stepping procedure [12].

The present method has been validated for the AGARD [1] CT5 standard test case. This is the case of unsteady transonic flow past an oscillating NACA0012 airfoil. In order to demonstrate the power of the method to handle multiple oscillating bodies, we compute flow past an oscillating pair of NACA0012 airfoils, one behind the other.

2 Least Squares Kinetic Upwind Method on Moving Nodes

Consider the 2-D Boltzmann equation

$$\frac{\partial F}{\partial t} + v_1 \frac{\partial F}{\partial x} + v_2 \frac{\partial F}{\partial y} = 0 \tag{1}$$

where F is the Maxwellian velocity distribution function, v_1 and v_2 are the Cartesian components of the molecular velocity. F in two dimensions is given by

$$F = \frac{\rho}{I_0}\frac{\beta}{\pi} \exp\left[-\beta(v_1-u_1)^2 - \beta(v_2-u_2)^2 - I/I_0\right] \tag{2}$$

where $\beta = 1/(2RT)$, I is internal energy variable, I_0 is the internal energy due to non-translational degrees of freedom, $I_0 = \frac{2-\gamma}{\gamma-1}RT$ and u_1 and u_2 are the Cartesian components of the fluid velocity, R is the gas constant and T is the absolute temperature of the fluid.

Now let w_1 and w_2 represent the Cartesian components of the velocity of any moving node. In order to deal with problems involving moving nodes, we define the derivative of F along the path of the node as

$$\left(\frac{dF}{dt}\right)_{mov} = \left(\frac{\partial F}{\partial t} + w_1 \frac{\partial F}{\partial x} + w_2 \frac{\partial F}{\partial y}\right)$$

using the above definition, the Boltzmann equation can be written

$$\left(\frac{dF}{dt}\right)_{mov} + \bar{v_1}\ \frac{\partial F}{\partial x} + \bar{v_2}\ \frac{\partial F}{\partial y} = 0 \tag{3}$$

where $\bar{v_1} = v_1 - w_1,\ \bar{v_2} = v_2 - w_2$ are the components of the particle velocity relative to the moving node. Using MCIR [10] splitting for both the components of molecular velocity, the Boltzmann Eq.(3) can be written as

$$\left(\frac{dF}{dt}\right)_{mov} + \frac{\bar{v}_1 + |\bar{v}_1|\phi_1}{2}\frac{\partial F}{\partial x} + \frac{\bar{v}_1 - |\bar{v}_1|\phi_1}{2}\frac{\partial F}{\partial x} + \frac{\bar{v}_2 + |\bar{v}_2|\phi_2}{2}\frac{\partial F}{\partial y} + \frac{\bar{v}_2 - |\bar{v}_2|\phi_2}{2}\frac{\partial F}{\partial y} = 0 \tag{4}$$

where $\phi_1,\ \phi_2$ are the dissipation control parameters corresponding to the two components of molecular velocity. The dissipation control parameters are conveniently chosen as $\phi_1 = \phi_2 = \Delta r^p$, where $0 < p < 1$ and Δr is the distance between a node and any point in its neighbourhood. We usually choose the closest point. The formal order of accuracy for this scheme is $(1+p)$, proof of this can be seen in [10, 11].

We define moment vector function Ψ by

$$\Psi = \left[1, v_1, v_2, I + \frac{v_1^2 + v_2^2}{2}\right]^T,$$

and define the Ψ moment as

$$\langle \Psi, F\rangle \equiv \int_0^\infty dI \int_{-\infty}^{\infty} dv_1 \int_{-\infty}^{\infty} dv_2\ \Psi F$$

Now the Ψ moment of the Eq.(4) will lead to the Modified Moving Kinetic Flux vector split Euler equations

$$\left(\frac{dU}{dt}\right)_{mov} + \frac{\partial}{\partial x}(GX_M^+) + \frac{\partial}{\partial x}(GX_M^-) + \frac{\partial}{\partial y}(GY_M^+) + \frac{\partial}{\partial y}(GY_M^-) = 0 \tag{5}$$

where U is the state vector given by $U = (\rho\ \ \rho u_1\ \ \rho u_2\ \ \rho e)^T$, $GX_M^{\pm}$ and $GY_M^{\pm}$ are the modified split fluxes for the moving nodes. The expressions for these modified moving split fluxes can be found in [8, 9]. The above equation is solved for the state vector using a dual time approach [12]. The various split flux derivatives are evaluated using a weighted least squares method [11], which is slightly different from that of Konark and Deshpande [6]. This approach reduces all the formulae to 1-D form. This is briefly explained in the next section. Flow tangency at the solid wall is imposed through the specular reflection model of kinetic theory [7]. The details of kinetic treatment of far field boundary condition can be seen in [9, 11]. MCIR splitting has been implemented in both these boundary treatments.

3 Weighted LSKUM_MN using Eigenvector Basis

The first order weighted least squares approximation [3] is given by the following equation,

$$A(w)\ \ \Delta F\ =\ b(w) \tag{6}$$

where

$$A(w) = \begin{bmatrix} \sum w_i \Delta {x_i}^2 & \sum w_i \Delta x_i \Delta y_i \\ \sum w_i \Delta x_i \Delta y_i & \sum w_i \Delta y_i^2 \end{bmatrix},\ \Delta F = \begin{bmatrix} F_x \\ F_y \end{bmatrix},\ b(w) = \begin{bmatrix} \sum w_i \Delta x_i \Delta F_i \\ \sum w_i \Delta y_i \Delta F_i \end{bmatrix}$$

w_i is the weight associated with each node i in the connectivity. The weights in the matrix $A(w)$ is now suitably chosen such that $A(w)$ gets diagonalised. The set of weights for each matrix $A(w)$ corresponding to each derivative can be conveniently chosen. However we should clearly recognise that a different set of weights are required to diagonalise each of the least squares matrix. The formulae for all the derivatives now reduce to 1-D form. This approach is referred to as the Weighted Least Squares using EigenVector basis.

4 Results and Discussion

We first present results for unsteady flow past NACA 0012 airfoil undergoing pitching oscillations about quarter chord. The flow conditions corresponds to the standard AGARD [1] test case. The free stream Mach number is $M = 0.755$. The oscillation cycle is defined by

$$\alpha(t) = \alpha_m + \alpha_0 sin(\omega t) \text{ where } \alpha_m = 0.016^0, \ \ \alpha_0 = 2.51^0,$$

where $\alpha(t)$ represents the instanteneous angle of attack, α_m represents the mean angle of attack. Reduced frequency based on chord length c of the airfoil is given by $k = \frac{\omega c}{2U_\infty}$, where ω represents the circular pitch frequency and U_∞ is the free stream fluid velocity. For the present test case $k = 0.0814$.

We have used point distribution containing 19200 points obtained from a structured grid. There are 320 points on the airfoil and 60 points in the normal direction. Farfield boundary is at about 10 chords distance from the airfoil. The whole set of points along with those on the airfoil oscillate about the quarter chord of the airfoil. The connectivity is obtained using a quadtree preprocessor [8].

For the unsteady computations a total of 81 real time steps are used in each cycle. The weight function $w_i = \frac{C_i}{d_i^{2n}}$ is used with $n = 1$, d_i is the distance of any connectivity point i and C_i is constant chosen such that the least squares matrix is diagonalised [11]. For the dissipation control parameter $\phi = (\Delta r)^p$, we use $p = 0.3$ and Δr is the distance of the closest connectivity point. For every real time step we require about 2000 iterations of pseudo time marching in the inner loop. The computations are done typically for about 2 to 3 cycles. Fig. 1 shows the comparison of the computed results with the experimental values for the normal force coefficient C_N and moment coefficient C_M. A good comparison is obtained. Fig. 2 shows the comparison of fourier components of surface pressure variations by the present computation with experiment. Again a good comparison has been obtained. The experimental results for this has been extracted from the results presented in [2].

In order to demonstrate the power of the method we consider unsteady flow past two oscillating NACA 0012 airfoils which are one behind the other. Fig.3 shows the distribution of points around this configuration. For this test case we consider two types of oscillation cycles. In the first type we consider both the airfoils oscillating in phase with the oscillation cycle as defined previously for the single airfoil test case. In the second type, the two airfoils oscillate with a phase difference of 180 degrees. The free stream flow conditions are also similar to the oscillating single airfoil test case. Connectivity at the end of each real time step is generated using a quad tree preprocessor [8].

Fig.4 shows the computed results for lift coefficient C_L and moment coefficient C_M for both front and back airfoil with respect to the angle of attack. These are the results for both the airfoils oscillating in phase. Even though we do not have any experimental results to compare, the results do show the effect of the front airfoil on the one behind it. This is evident from the plots. A similar set of results can be seen in Fig.5 for the case when the two airfoils are oscillating with a phase shift of 180^o. In this case the C_L loop for the airfoil behind is more wider compared to the one in the front. This is just the opposite behaviour for the case of oscillation in phase.

5 Conclusions

A lot of new developments in the grid free method have been implemeted in the present work for computations of unsteady flow past oscillating bodies. Single step higher order accuracy has been achieved through the MCIR splitting. Robustness of least squares formulae for derivatives has been achieved by use of collapsed 1-D formulae. It is possible to reduce 2-D form to 1-D form by a suitable choice of weights. All these have been implemented in the LSKUM_MN formulation. Further the unsteady caluculations are done using the dual time stepping strategy. The grid free method LSKUM_MN with all these features included has been demonstrated to work on the standard AGARD test case of oscillating NACA 0012 airfoil. Further to demonstrate the power of the method we have computed the unsteady flow past two independenlty oscillating NACA 0012 airfoils. In the near future we hope to apply our new tool TKFMG(Turbomachinery KFvs for Moving Grids) code to compute unsteady flows through the oscillating turbomachinery blades.

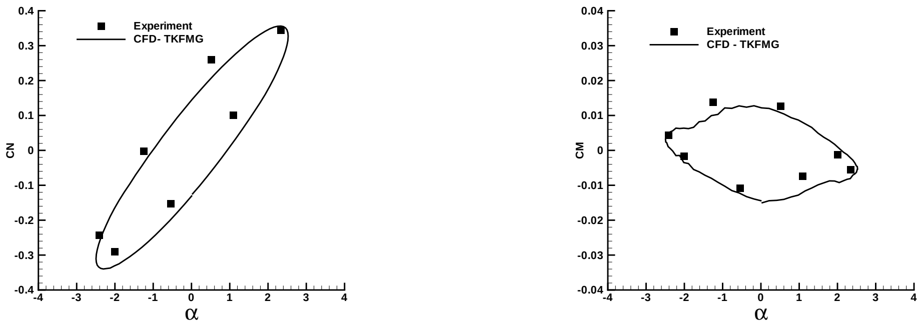

Figure 1: Comparison of lift and moment coefficents

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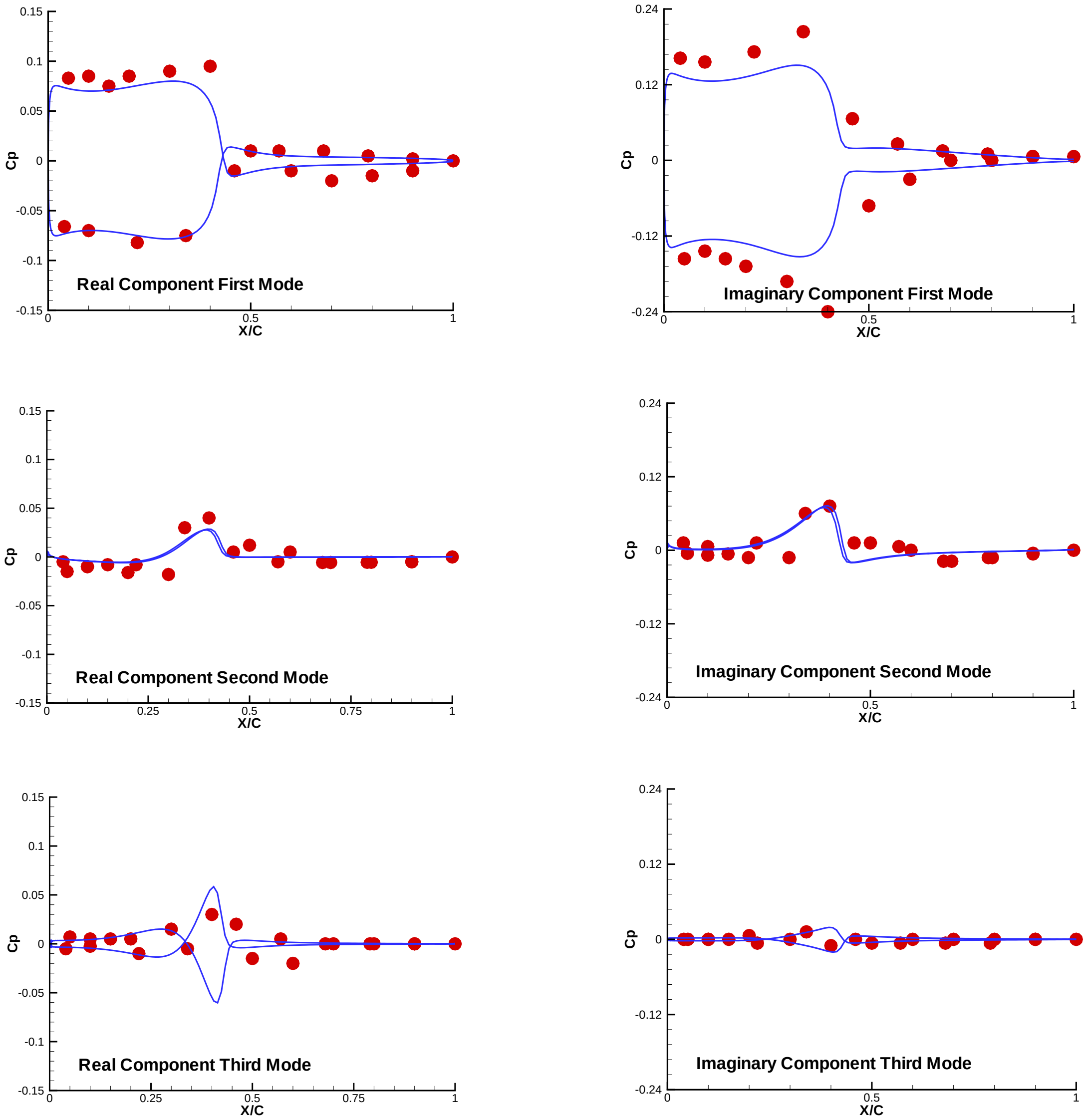

Figure 2: Comparison of Fourier components of surface pressure variations

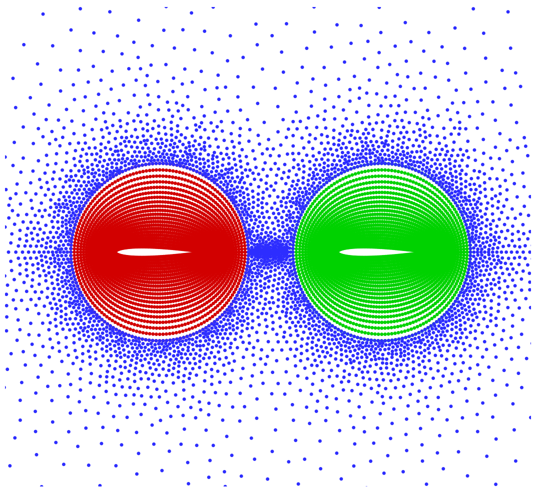

Figure 3: Points distributions for the two airfoil configuration

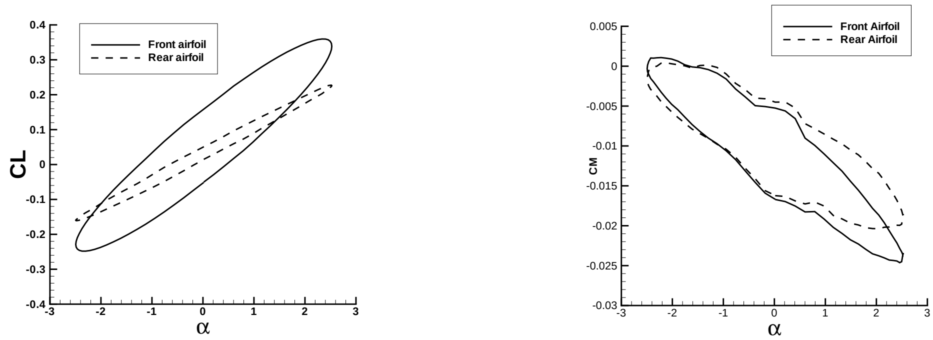

Figure 4: Two airfoil: phase shift $= 0^o$: Variation of C_L C_M with α

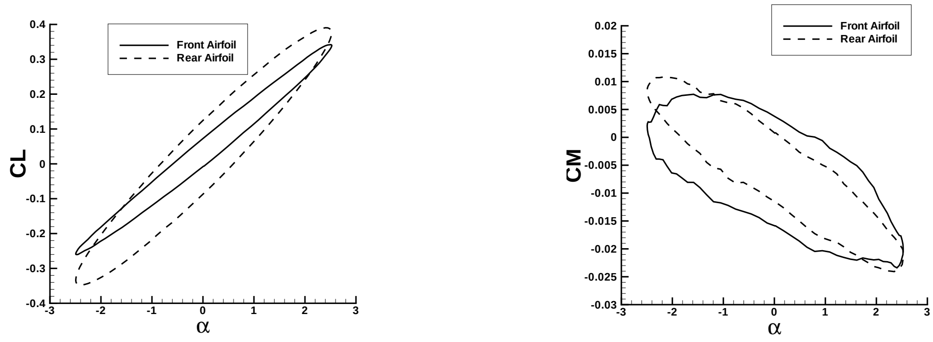

Figure 5: Two airfoil: phase shift $= 180^o$: Variation of C_L C_M with α